FLOATING POINT

CS 045

Computer Organization and Architecture

Prof. Donald J. Patterson

Adapted from Bryant and O'Hallaron,
Computer Systems:
A Programmer's Perspective, Third Edition

- BACKGROUND:
  - FRACTIONAL BINARY NUMBERS
- IEEE FLOATING POINT STANDARD:
  - DEFINITION
- EXAMPLE AND PROPERTIES
- ROUNDING, ADDITION, MULTIPLICATION
- FLOATING POINT IN C
- SUMMARY

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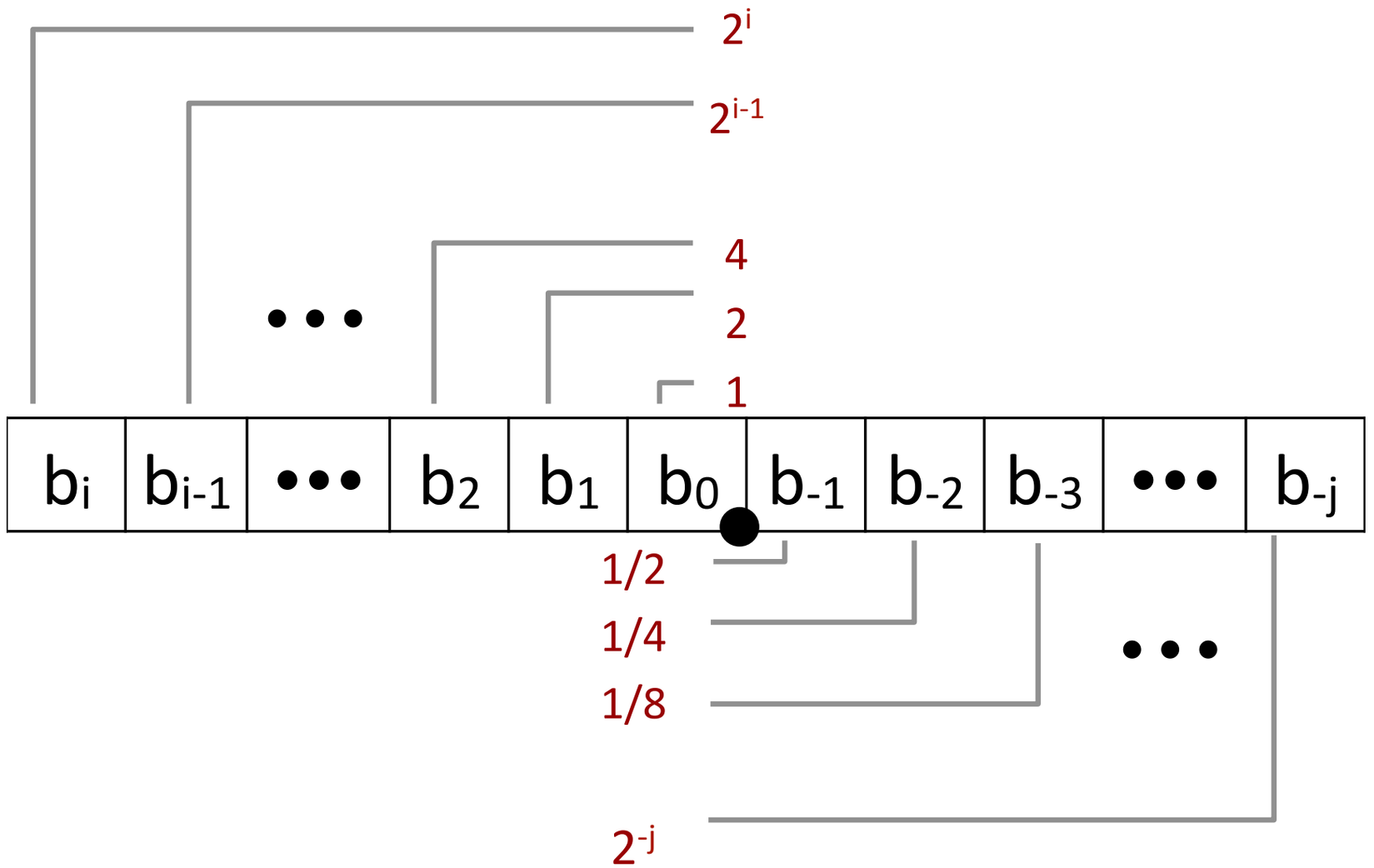
What is

1011.101_2

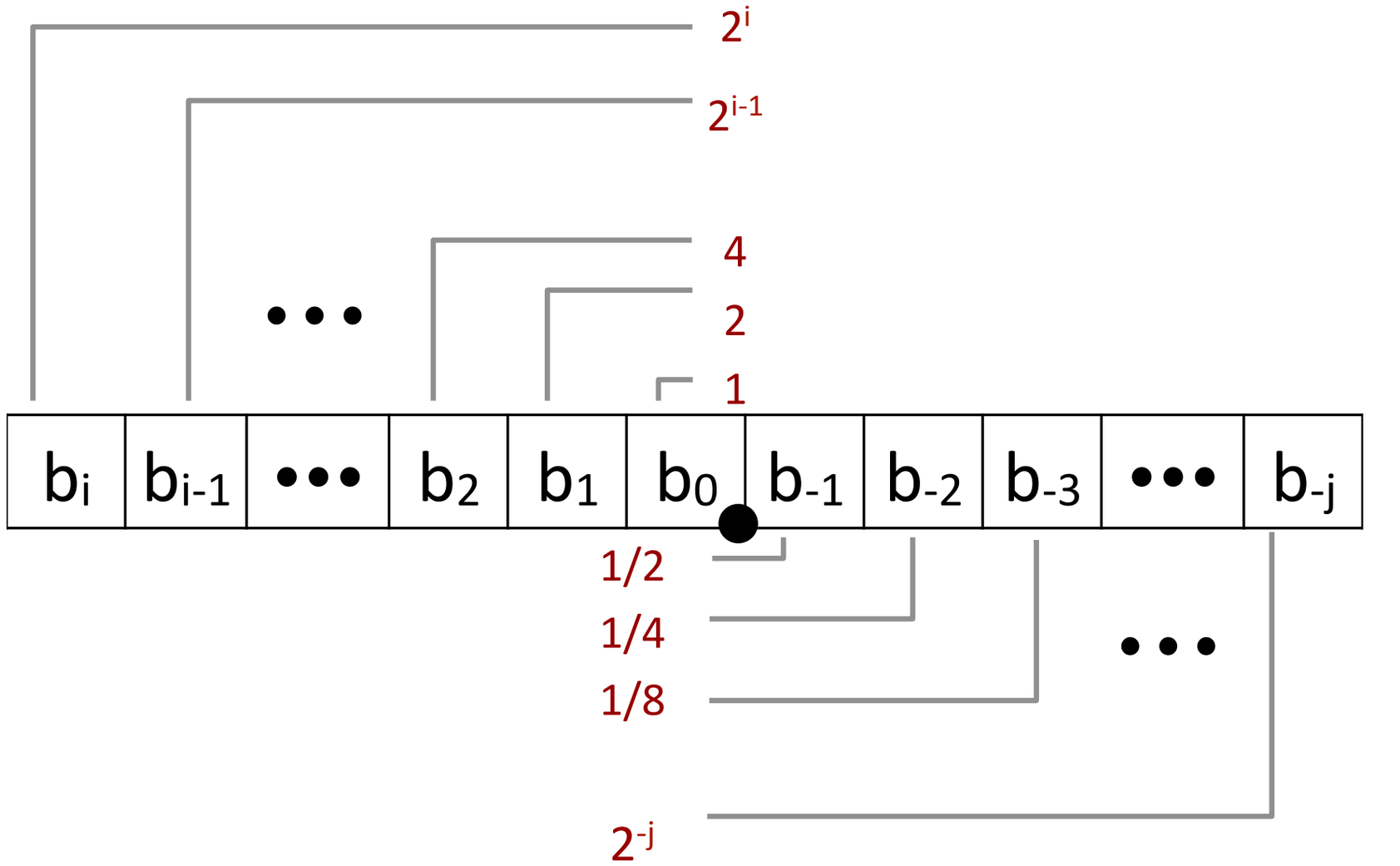
?

FRACTIONAL BINARY NUMBERS



- Representation
  - Bits to right of "binary point" represent fractional powers of 2
- Represents rational number: $\sum_{k=-j}^{i} (b_k \times 2^k)$

FRACTIONAL BINARY NUMBERS



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- Representation
  - Bits to right of “binary point” represent fractional powers of 2
- Represents rational number:

FRACTIONAL BINARY NUMBERS

EXAMPLES

- Value — Representation
  - 5 3/4 — 101.11_2
  - 2 7/8 — 010.111_2
  - 1 7/16 — 001.0111_2

- Observations
  - Divide by 2 by shifting right (unsigned)
  - Multiply by 2 by shifting left
  - Numbers of form $0.111111\ldots_2$ are just below 1.0
    - $1/2 + 1/4 + 1/8 + \ldots + 1/2^i + \ldots \rightarrow 1.0$
    - Use notation $1.0 - \varepsilon$

FRACTIONAL BINARY NUMBERS

REPRESENTABLE NUMBERS

- Limitation #1
  - Can only exactly represent numbers of the form $x/2^k$
    - Other rational numbers have repeating bit representations
      - Value Representation
      - 1/3 $0.0101010101[01]\ldots_2$
      - 1/5 $0.001100110011[0011]\ldots_2$
      - 1/10 $0.0001100110011[0011]\ldots_2$

FRACTIONAL BINARY NUMBERS

REPRESENTABLE NUMBERS

- Limitation #2
  - Just one setting of binary point within the w bits
    - Limited range of numbers (very small values? very large?)

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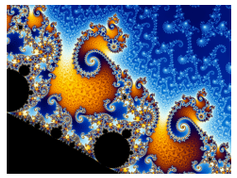
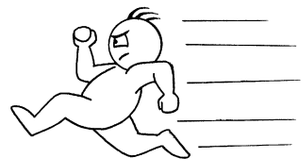
IEEE FLOATING POINT

- IEEE Standard 754
  - Established in 1985 as uniform standard for floating point arithmetic
  - Before that, many idiosyncratic formats
  - Supported by all major CPUs

- Driven by numerical concerns
  - Nice standards for rounding, overflow, underflow
  - Hard to make fast in hardware
  - Numerical analysts predominated over hardware designers in defining standard





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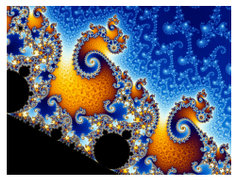

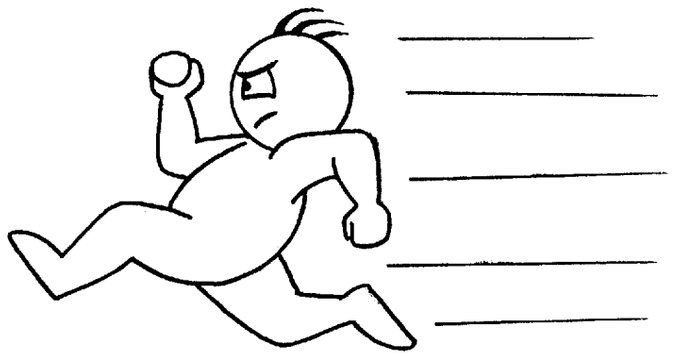

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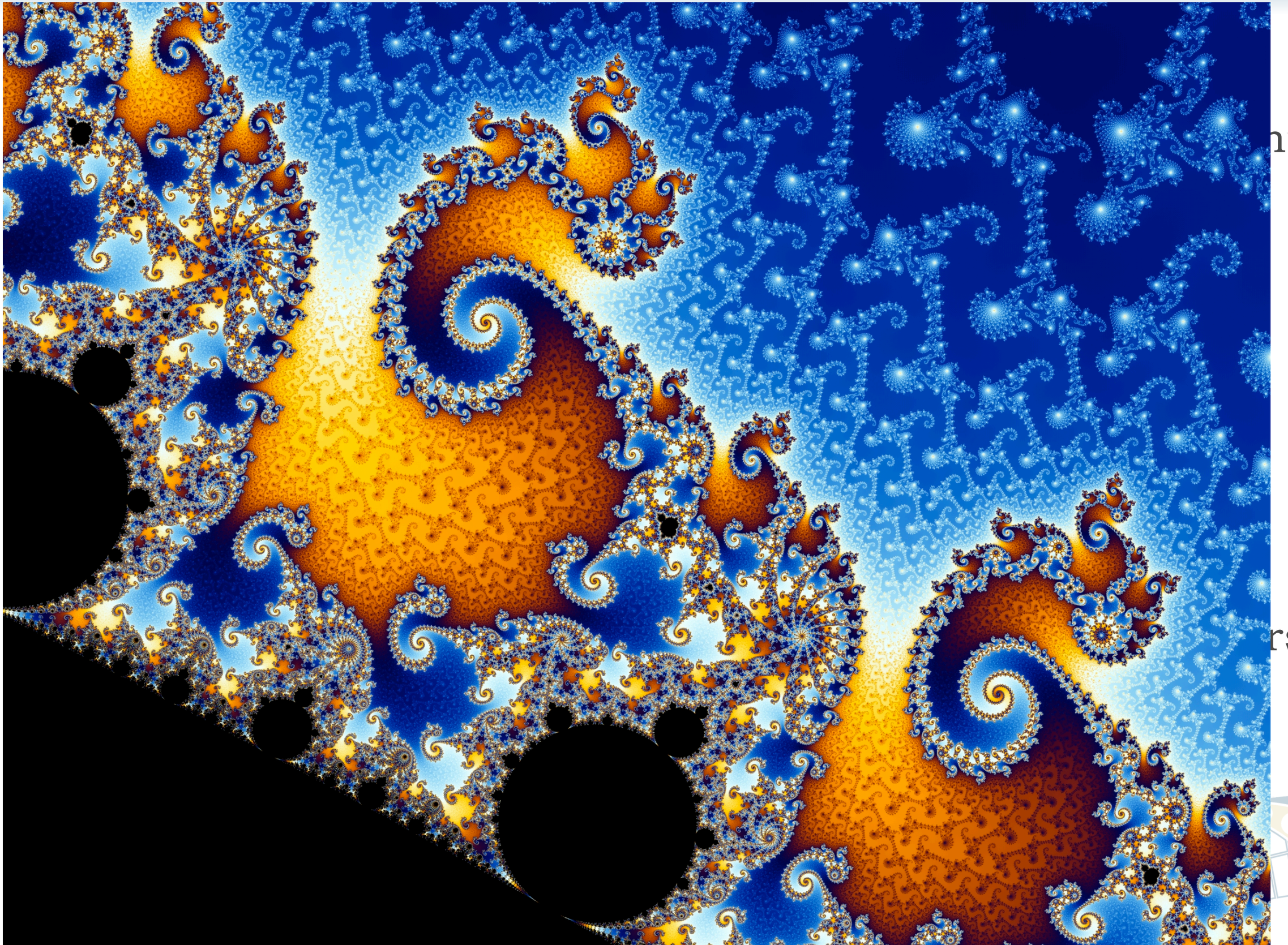
- 

IEEE FLOATING POINT

FLOATING POINT REPRESENTATION

- Numerical Form:

$$(-1)^{s}\ M\ \ 2^{E}$$

  - Sign bit s determines whether number is negative or positive
  - Significand M normally a fractional value in range [0.0,2.0).
  - Exponent E weights value by power of two

- Encoding
  - MSB s is sign bit s
  - exp field encodes E (but is not equal to E)
  - frac field encodes M (but is not equal to M)

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IEEE FLOATING POINT

PRECISION OPTIONS

- Single precision: 32 bits
- Double precision: 64 bits
- Extended precision: 80 bits (Intel only)

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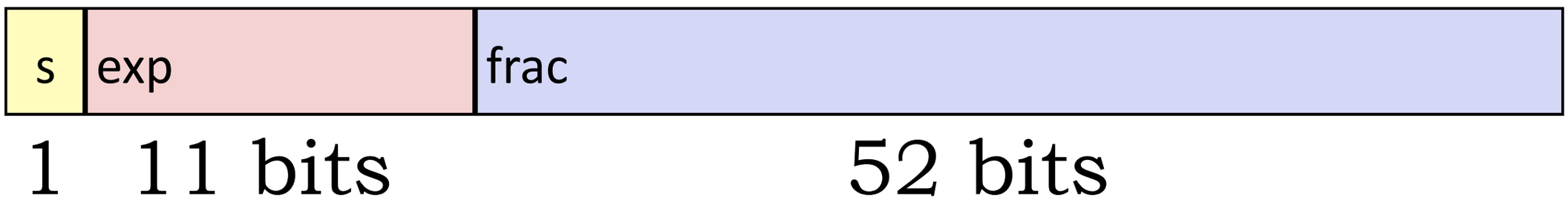

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IEEE FLOATING POINT

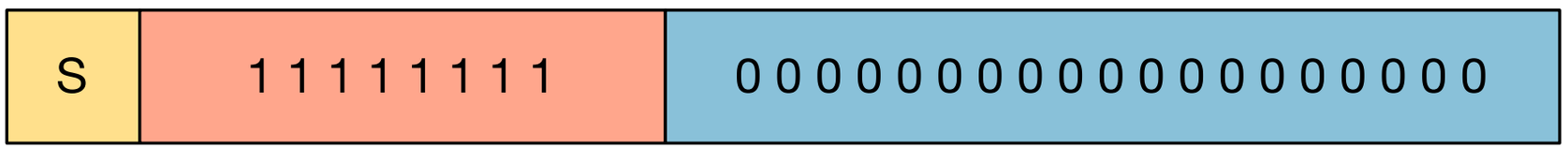
3 CASES

- Normalized

- Denormalized

- Special cases

∞



NaN

IEEE FLOATING POINT

"NORMALIZED" VALUES

$$(-1)^s \; M \;\; 2^E$$

- When: exp ≠ 000…0 and exp ≠ 111…1

- Exponent coded as a biased value: E = Exp – Bias
  - Exp: unsigned value of exp field
  - Bias = $2^{k-1} - 1$, where k is number of exponent bits
    - Single precision: 127 (Exp: 1…254, E: -126…127)
    - Double precision: 1023 (Exp: 1…2046, E: -1022…1023)

- Significand coded with implied leading 1: M = $1.xxx…x_2$
  - xxx…x: bits of frac field
  - Minimum when frac=000…0 (M = 1.0)
  - Maximum when frac=111…1 (M = 2.0 – ε)
  - Get extra leading bit for "free"

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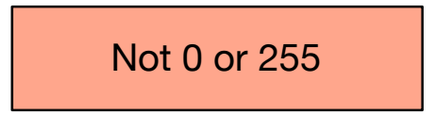

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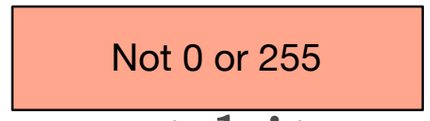

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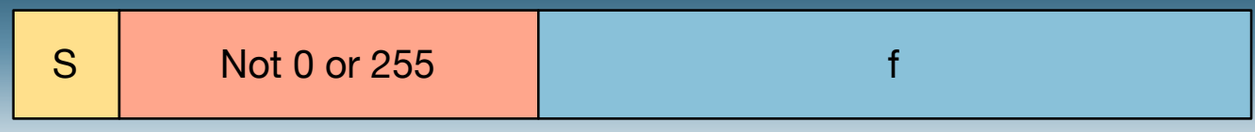

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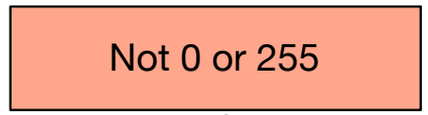

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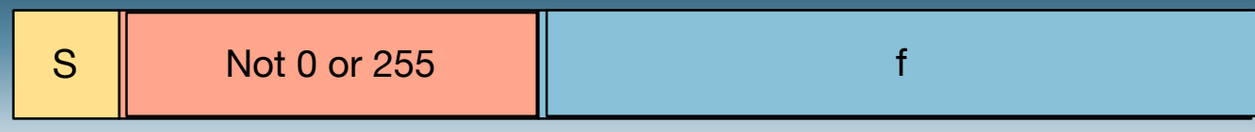

DENORMALIZED VALUES

$$(-1)^s \; M \;\; 2^E$$

- Condition: exp = 000…0

- Exponent value: E = 1 – Bias (instead of Exp – Bias)
- Significand coded with implied leading 0: M = 0.xxx…x_2
  - xxx…x: bits of frac

- Cases
  - exp = 000…0, frac = 000…0
    - Represents zero value
    - Note distinct values: +0 and –0 (why?)
  - exp = 000…0, frac ≠ 000…0
    - Numbers closest to 0.0
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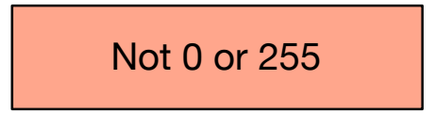

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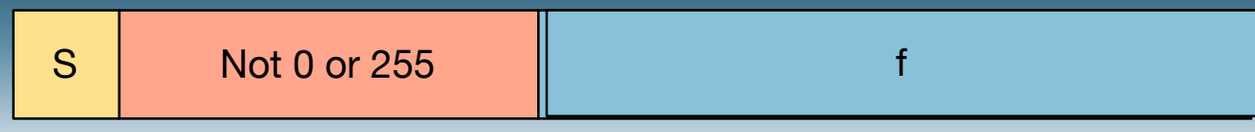

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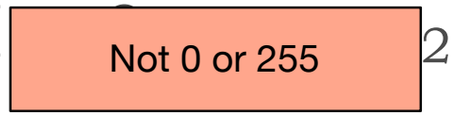

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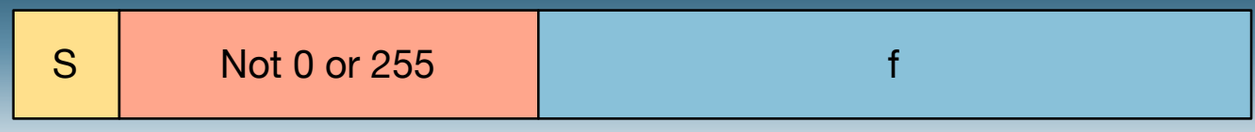

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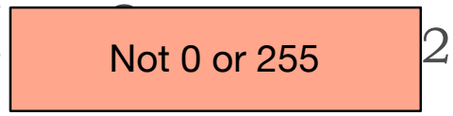

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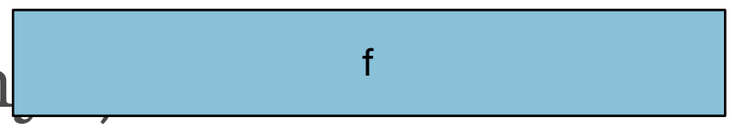

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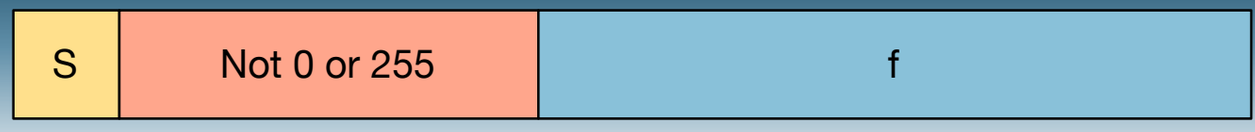


SPECIAL VALUES

$$(-1)^s \; M \;\; 2^E$$

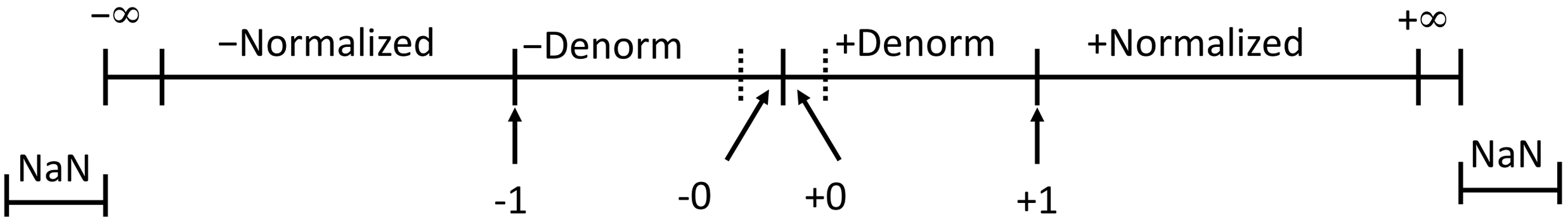
- Condition: exp = 111…1

- Case: exp = 111…1, frac = 000…0
  - Represents value ∞ (infinity)
  - Operation that overflows
  - Both positive and negative
  - E.g., 1.0/0.0 = −1.0/−0.0 = + ∞, 1.0/−0.0 = − ∞

- Case: exp = 111…1, frac ≠ 000…0
  - Not-a-Number (NaN)
  - Represents case when no numeric value can be determined
  - E.g., sqrt(−1), ∞ − ∞, ∞ *0

IEEE FLOATING POINT

VISUALIZATION: FLOATING POINT ENCODINGS



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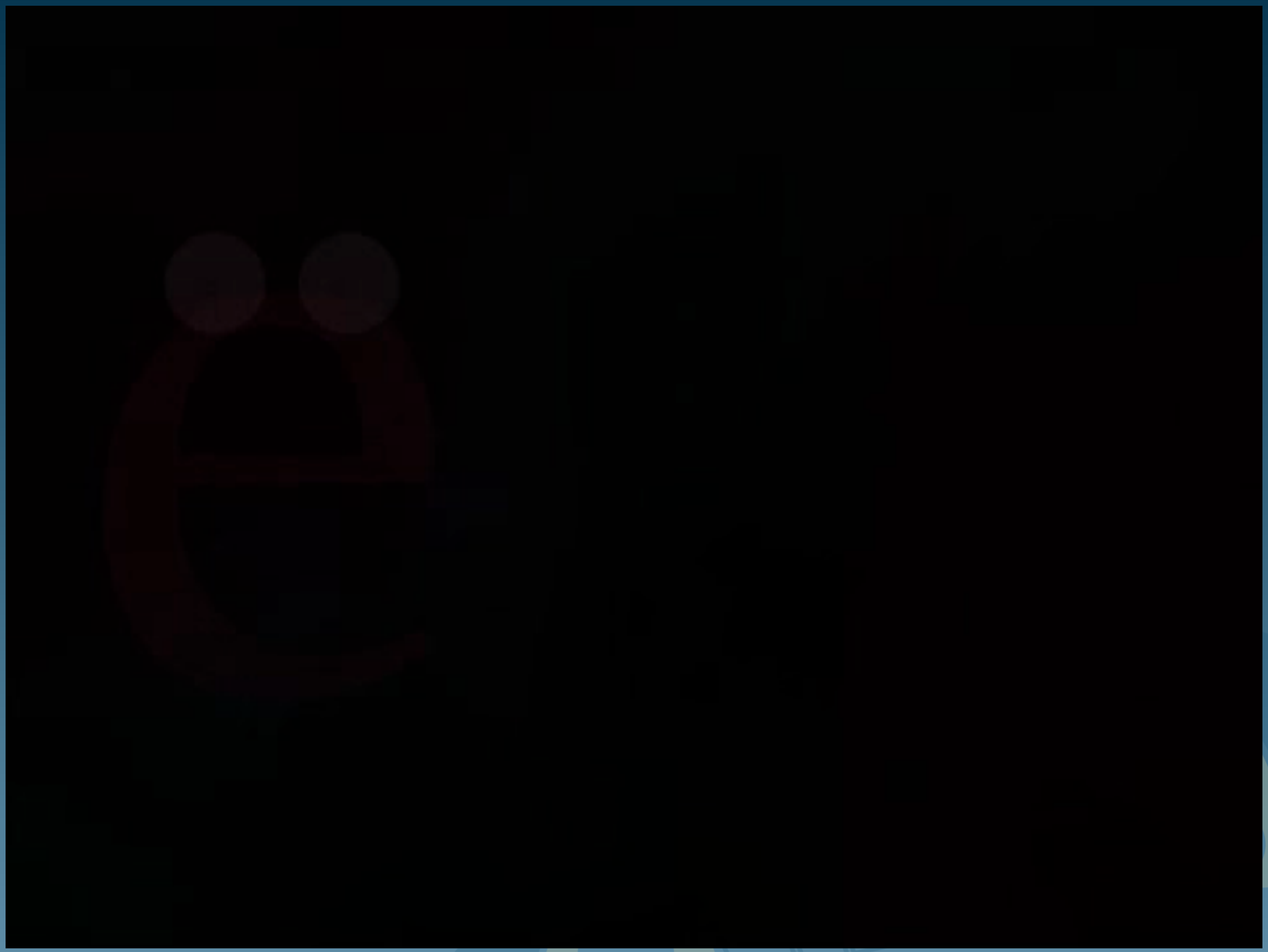

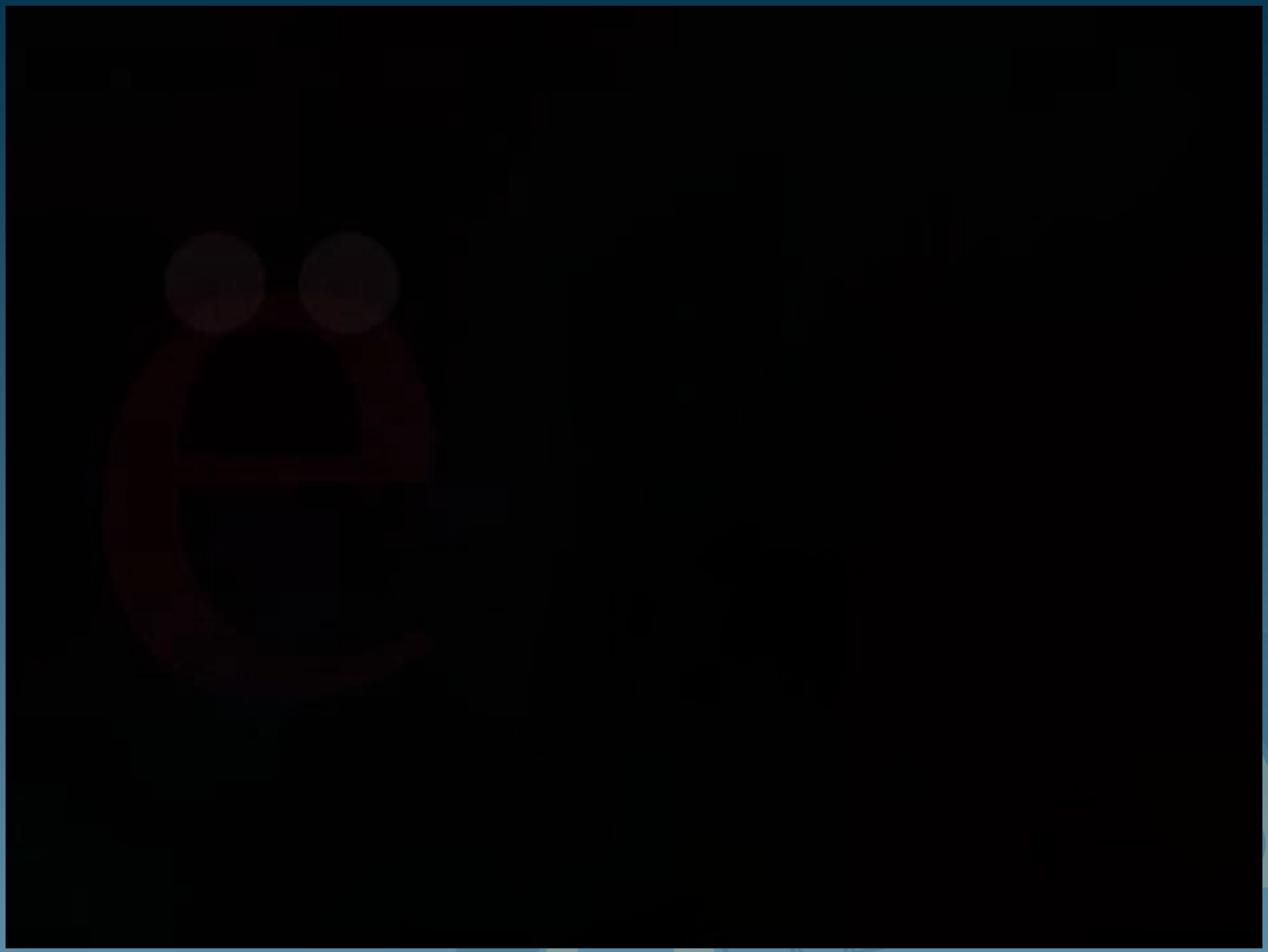

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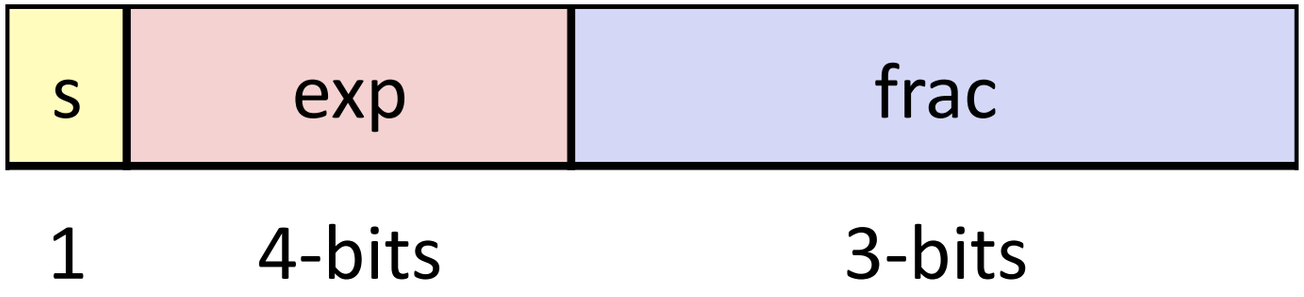
TINY FLOATING POINT EXAMPLE



- 8-bit Floating Point Representation
  - the sign bit is in the most significant bit
  - the next four bits are the exponent, with a bias of 7
  - the last three bits are the frac

- Same general form as IEEE Format
  - normalized, denormalized
  - representation of 0, NaN, infinity

DYNAMIC RANGE (POSITIVE ONLY)

s	exp	frac	E	Value
0	0000	000	-6	0
0	0000	001	-6	1/8*1/64 = 1/512
0	0000	010	-6	2/8*1/64 = 2/512
…				
0	0000	110	-6	6/8*1/64 = 6/512
0	0000	111	-6	7/8*1/64 = 7/512
0	0001	000	-6	8/8*1/64 = 8/512
0	0001	001	-6	9/8*1/64 = 9/512
…				
0	0110	110	-1	14/8*1/2 = 14/16
0	0110	111	-1	15/8*1/2 = 15/16
0	0111	000	0	8/8*1 = 1
0	0111	001	0	9/8*1 = 9/8
0	0111	010	0	10/8*1 = 10/8
…				
0	1110	110	7	14/8*128 = 224
0	1110	111	7	15/8*128 = 240
0	1111	000	n/a	inf

DYNAMIC RANGE (POSITIVE ONLY)

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…				
0	0000	110	-6	6/8*1/64 = 6/512
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0	0001	000	-6	8/8*1/64 = 8/512
0	0001	001	-6	9/8*1/64 = 9/512
…				
0	0110	110	-1	14/8*1/2 = 14/16
0	0110	111	-1	15/8*1/2 = 15/16
0	0111	000	0	8/8*1 = 1
0	0111	001	0	9/8*1 = 9/8
0	0111	010	0	10/8*1 = 10/8
…				
0	1110	110	7	14/8*128 = 224
0	1110	111	7	15/8*128 = 240
0	1111	000	n/a	inf

DYNAMIC RANGE (POSITIVE ONLY)

	s	exp	frac	E	Value
Denormalized numbers	0	0000	000	-6	0
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	…				
	0	0000	110	-6	6/8*1/64 = 6/512
	0	0000	111	-6	7/8*1/64 = 7/512
	0	0001	000	-6	8/8*1/64 = 8/512
	0	0001	001	-6	9/8*1/64 = 9/512
	…				
	0	0110	110	-1	14/8*1/2 = 14/16
	0	0110	111	-1	15/8*1/2 = 15/16
	0	0111	000	0	8/8*1 = 1
	0	0111	001	0	9/8*1 = 9/8
	0	0111	010	0	10/8*1 = 10/8
	…				
	0	1110	110	7	14/8*128 = 224
	0	1110	111	7	15/8*128 = 240
	0	1111	000	n/a	inf

DYNAMIC RANGE (POSITIVE ONLY)

	s exp frac	E	Value	
Denormalized numbers	0 0000 000	-6	0	**closest to zero**
	0 0000 001	-6	1/8*1/64 = 1/512	
	0 0000 010	-6	2/8*1/64 = 2/512	
	…			
	0 0000 110	-6	6/8*1/64 = 6/512	
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	0 0001 000	-6	8/8*1/64 = 8/512	
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	0 0110 110	-1	14/8*1/2 = 14/16	
	0 0110 111	-1	15/8*1/2 = 15/16	
	0 0111 000	0	8/8*1 = 1	
	0 0111 001	0	9/8*1 = 9/8	
	0 0111 010	0	10/8*1 = 10/8	
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	0	0000	111	-6	7/8*1/64 = 7/512	**largest denorm**
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Normalized numbers	0	0001	000	-6	8/8*1/64 = 8/512	**smallest norm**
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	0 0110 110	-1	14/8*1/2 = 14/16	
	0 0110 111	-1	15/8*1/2 = 15/16	**closest to 1 below**
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INTERESTING NUMBERS

Description	exp	frac	Numeric Value {single,double}
• Zero	00…00	00…00	0.0
• Smallest Pos. Denorm.	00…00	00…01	$2^{-\{23,52\}} \times 2^{-\{126,1022\}}$
• Single ≈ 1.4×10^{-45}			
• Double ≈ 4.9×10^{-324}			
• Largest Denormalized	00…00	11…11	$(1.0 - \varepsilon) \times 2^{-\{126,1022\}}$
• Single ≈ 1.18×10^{-38}			
• Double ≈ 2.2×10^{-308}			
• Smallest Pos. Normalized	00…01	00…00	$1.0 \times 2^{-\{126,1022\}}$
• Just larger than largest denormalized			
• One	01…11	00…00	1.0
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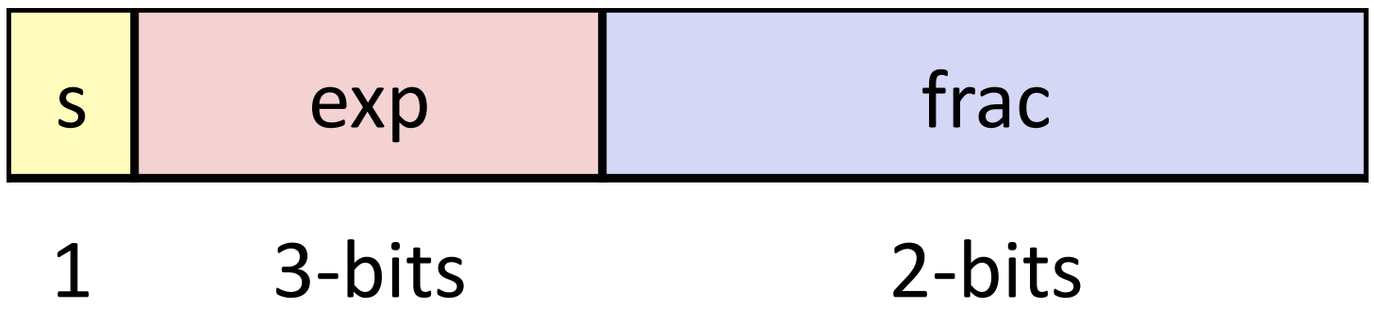
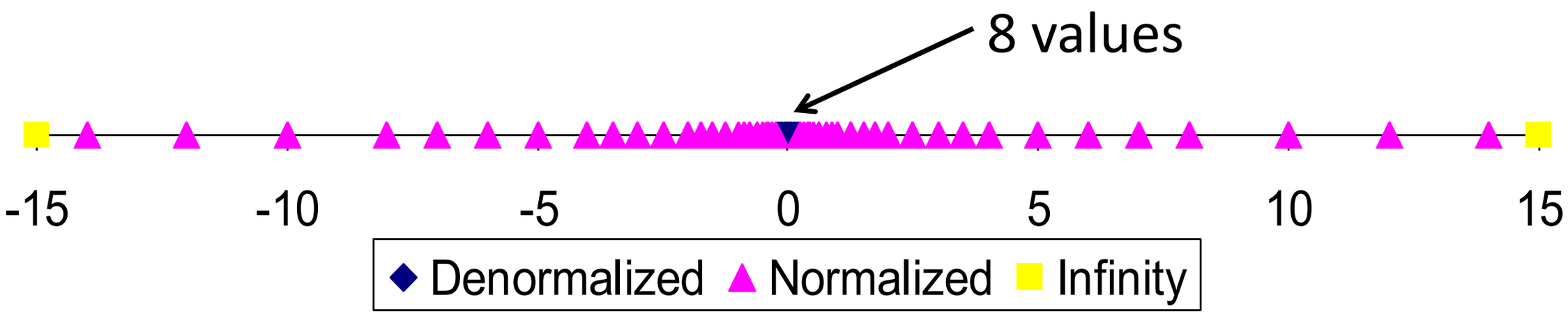
DISTRIBUTION OF VALUES

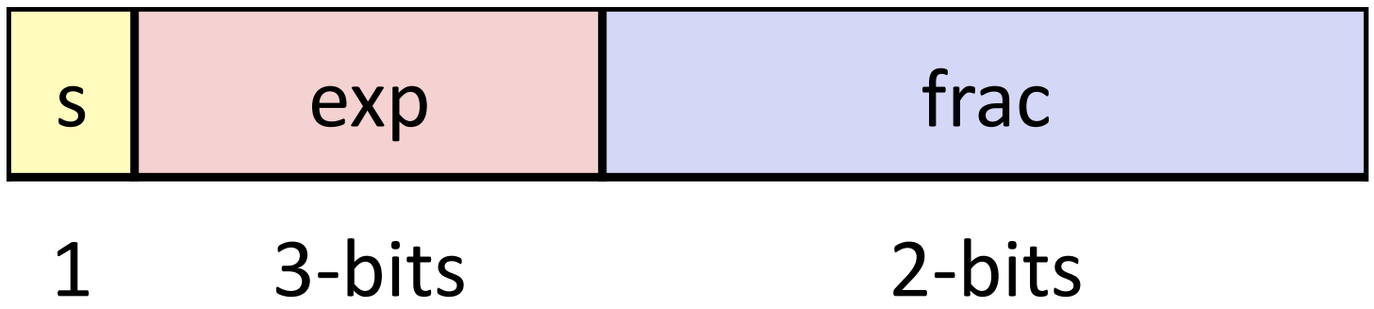


- 6-bit IEEE-like format
  - e = 3 exponent bits
  - f = 2 fraction bits
  - Bias is $2^{3-1}-1 = 3$

- Notice how the distribution gets denser toward zero.



DISTRIBUTION OF VALUES (CLOSE-UP VIEW)



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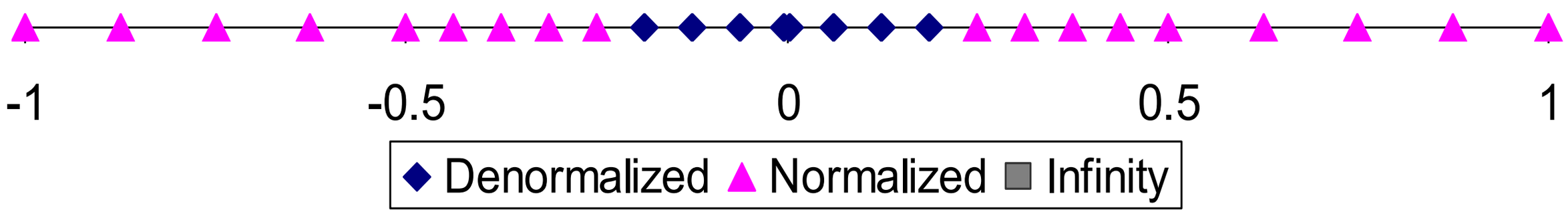

SPECIAL PROPERTIES OF THE IEEE ENCODING

- FP Zero Same as Integer Zero
  - All bits = 0

- Can (Almost) Use Unsigned Integer Comparison
  - Must first compare sign bits
  - Must consider –0 = 0
  - NaNs problematic
    - Will be greater than any other values
    - What should comparison yield?
  - Otherwise OK
    - Denorm vs. normalized
    - Normalized vs. infinity

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- BACKGROUND:
  - FRACTIONAL BINARY NUMBERS
- IEEE FLOATING POINT STANDARD:
  - DEFINITION
- EXAMPLE AND PROPERTIES
- ROUNDING, ADDITION, MULTIPLICATION
- FLOATING POINT IN C
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ROUNDING, ADDITION, MULTIPLICATION

FLOATING POINT OPERATIONS: BASIC IDEA

- $x +_f y = \text{Round}(x + y)$
- $x *_f y = \text{Round}(x * y)$
- Basic idea
  - First compute exact result
  - Make it fit into desired precision
    - Possibly overflow if exponent too large
    - Possibly round to fit into frac

ROUNDING, ADDITION, MULTIPLICATION

ROUNDING

- Rounding Modes (illustrate with $ rounding)

	$1.40	$1.60	$1.50	$2.50	–$1.50
Towards zero	$1	$1	$1	$2	–$1
Round down (–)	$1	$1	$1	$2	–$2
Round up (+)	$2	$2	$2	$3	–$1
Nearest Even (default)	$1	$2	$2	$2	–$2

ROUNDING, ADDITION, MULTIPLICATION

CLOSER LOOK AT ROUND-TO-EVEN

- Default Rounding Mode
  - Hard to get any other kind without dropping into assembly
  - All others are statistically biased
    - Sum of set of positive numbers will consistently be over- or under- estimated

- Applying to Other Decimal Places / Bit Positions
  - When exactly halfway between two possible values
    - Round so that least significant digit is even
  - E.g., round to nearest hundredth
  - 7.8949999 7.89 (Less than half way)
  - 7.8950001 7.90 (Greater than half way)
  - 7.8950000 7.90 (Half way—round up)
  - 7.8850000 7.88 (Half way—round down)

ROUNDING, ADDITION, MULTIPLICATION

ROUNDING BINARY NUMBERS

- Binary Fractional Numbers
  - "Even" when least significant bit is 0
  - "Half way" when bits to right of rounding position = $100\ldots_2$

- Examples
  - Round to nearest 1/4 (2 bits right of binary point)
  - Value Binary Rounded Action Rounded Value
    - 2 3/32 10.00011_2 10.00_2 (<1/2—down) 2
    - 2 3/16 10.00110_2 10.01_2 (>1/2—up) 2 1/4
    - 2 7/8 10.11100_2 11.00_2 (1/2—up) 3
    - 2 5/8 10.10100_2 10.10_2 (1/2—down) 2 1/2

ROUNDING, ADDITION, MULTIPLICATION

FP MULTIPLICATION

- $(-1)^{s1}\ M1\ \ 2^{E1}\ \ x\ \ (-1)^{s2}\ M2\ \ 2^{E2}$
- Exact Result: $(-1)^{s}\ M\ \ 2^{E}$
  - Sign s: $s1\ \hat{}\ s2$
  - Significand M: $M1 \times M2$
  - Exponent E: $E1 + E2$

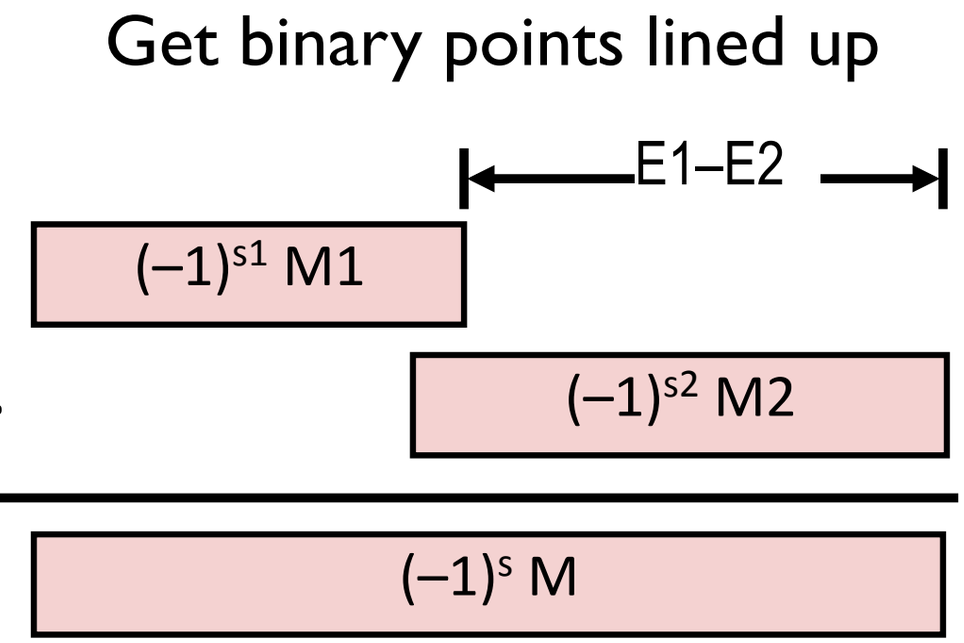
- "Normalizing"
  - If $M \geq 2$, shift M right, increment E
  - If E out of range, overflow
  - Round M to fit frac precision

- Implementation
  - Biggest chore is multiplying significands

ROUNDING, ADDITION, MULTIPLICATION

FLOATING POINT ADDITION

- $(-1)^{s1}$ M1 2^{E1} x $(-1)^{s2}$ M2 2^{E2}
  - Assume E1 > E2
- Exact Result: $(-1)^{s}$ M 2^{E}
  - Sign s, significand M:
    - Result of signed align & add
  - Exponent E: E1



- Fixing
  - If M ≥ 2, shift M right, increment E
  - if M < 1, shift M left k positions, decrement E by k
  - Overflow if E out of range
  - Round M to fit frac precision

ROUNDING, ADDITION, MULTIPLICATION

MATHEMATICAL PROPERTIES OF FP ADD

- Compare to those of Abelian Group
  - Closed under addition?
    - But may generate infinity or NaN
  - Commutative?
  - Associative?
    - Overflow and inexactness of rounding
    - (3.14+1e10)-1e10 = 0, 3.14+(1e10-1e10) = 3.14
- 0 is additive identity?
- Every element has additive inverse?
  - Yes, except for infinities & NaNs
- Monotonicity
  - $a \geq b \Rightarrow a+c \geq b+c$?
    - Except for infinities & NaNs

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  - Every element has additive inverse? Almost
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    - $a \geq b \Rightarrow a+c \geq b+c$?
      - Except for infinities & NaNs

ROUNDING, ADDITION, MULTIPLICATION

MATHEMATICAL PROPERTIES OF FP ADD

- Compare to those of Abelian Group
  - Closed under addition? Yes
    - But may generate infinity or NaN
  - Commutative? Yes
  - Associative? No
    - Overflow and inexactness of rounding
    - (3.14+1e10)-1e10 = 0, 3.14+(1e10-1e10) = 3.14
  - 0 is additive identity? Yes
  - Every element has additive inverse? Almost
    - Yes, except for infinities & NaNs
  - Monotonicity
    - $a \geq b \Rightarrow a+c \geq b+c$? Almost
      - Except for infinities & NaNs

ROUNDING, ADDITION, MULTIPLICATION

MATHEMATICAL PROPERTIES OF FP MULT

- Compare to Commutative Ring
  - Closed under multiplication?
    - But may generate infinity or NaN
  - Multiplication Commutative?
  - Multiplication is Associative?
    - Possibility of overflow, inexactness of rounding
    - Ex: (1e20*1e20)*1e-20= inf, 1e20*(1e20*1e-20)= 1e20
  - 1 is multiplicative identity?
  - Multiplication distributes over addition?
    - Possibility of overflow, inexactness of rounding
    - 1e20*(1e20-1e20)= 0.0, 1e20*1e20 – 1e20*1e20 = NaN
- Monotonicity
  - $a \geq b$ & $c \geq 0 \Rightarrow a * c \geq b * c$?
    - Except for infinities & NaNs

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- IEEE FLOATING POINT STANDARD:
  - DEFINITION
- EXAMPLE AND PROPERTIES
- ROUNDING, ADDITION, MULTIPLICATION
- FLOATING POINT IN C
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FLOATING POINT IN C

- C Guarantees Two Levels
  - `float` single precision
  - `double` double precision

FLOATING POINT IN C

- Conversions/Casting
  - Casting between `int`, `float`, and `double` changes bit representation
  - `double/float` → `int`
    - Truncates fractional part
    - Like rounding toward zero
    - Not defined when out of range or NaN: Generally sets to TMin
  - `int` → `double`
    - Exact conversion, as long as int has ≤ 53 bit word size
  - `int` → `float`
    - Will round according to rounding mode

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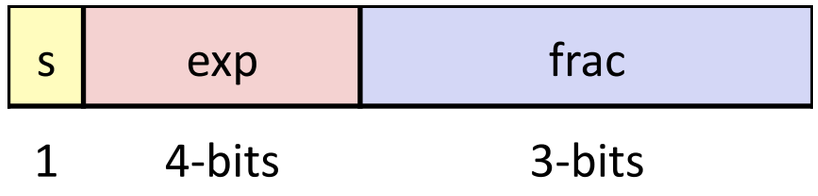
SUMMARY

FLOATING POINT IN C

- IEEE Floating Point has clear mathematical properties
  - Represents numbers of form M x 2^E
- One can reason about operations independent of implementation
  - As if computed with perfect precision and then rounded
- Not the same as real arithmetic
  - Violates associativity/distributivity
  - Makes life difficult for compilers & serious numerical applications programmers

WORK IT OUT

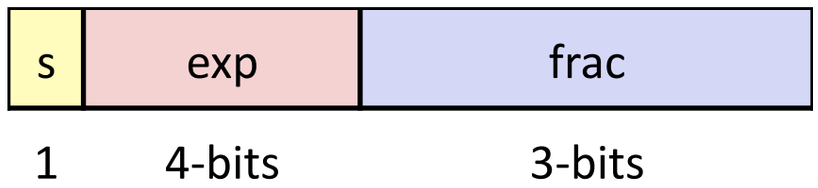
CREATING FLOATING POINT NUMBER



- Steps
  - Normalize to have leading 1
  - Round to fit within fraction
  - Postnormalize to deal with effects of rounding
- Case Study
  - Convert 8-bit unsigned numbers to tiny floating point format
  - Example Numbers
    - 128 10000000
    - 15 00001101
    - 33 00010001
    - 35 00010011
    - 138 10001010
    - 63 00111111

CREATING FLOATING POINT NUMBER

NORMALIZE

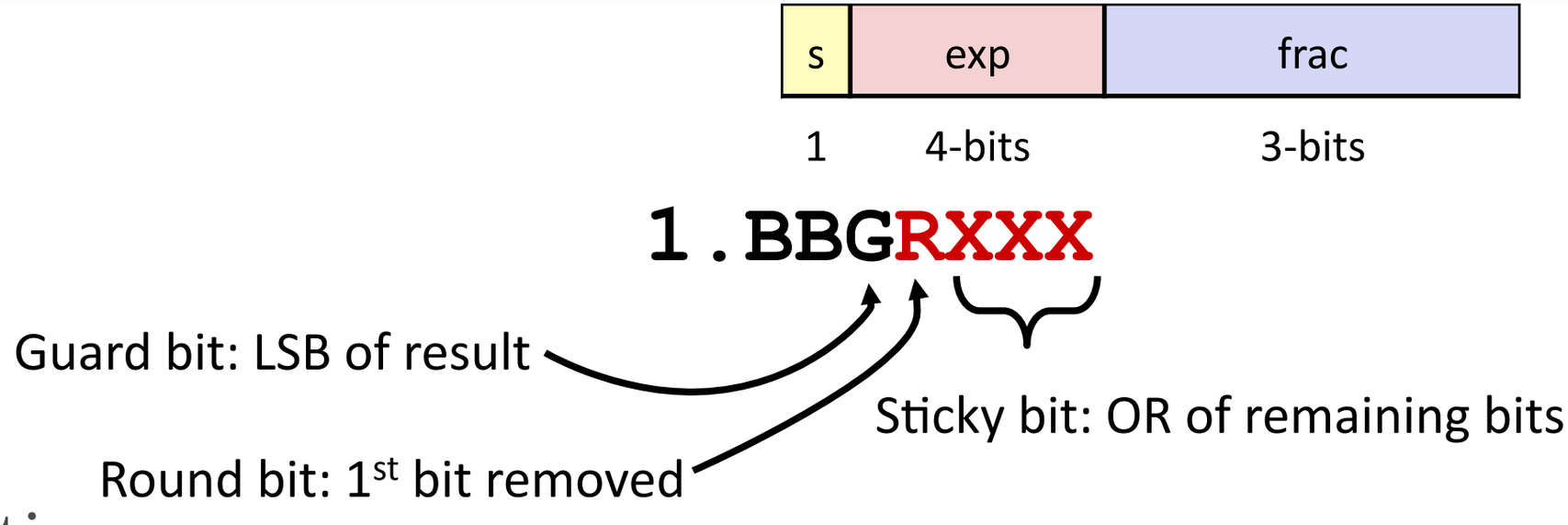


- Requirement
  - Set binary point so that numbers of form 1.xxxxx
  - Adjust all to have leading one
    - Decrement exponent as shift left

Value	Binary	Fraction	Exponent
128	10000000	1.0000000	7
15	00001101	1.1010000	3
17	00010001	1.0001000	4
19	00010011	1.0011000	4
138	10001010	1.0001010	7
63	00111111	1.1111100	5

CREATING FLOATING POINT NUMBER

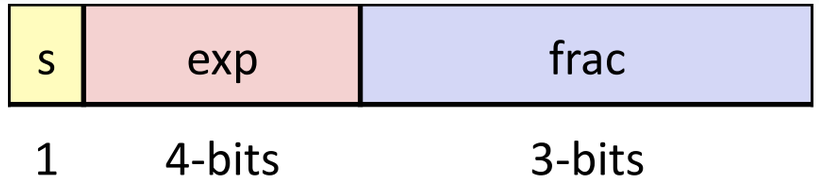
ROUNDING



- Round up conditions
- Round = 1, Sticky = 1 → > 0.5
- Guard = 1, Round = 1, Sticky = 0 → Round to even
  - Value Fraction GRS Incr? Rounded
  - 128 1.00000000 000 N 1.000
  - 15 1.10100000 100 N 1.101
  - 17 1.00011000 010 N 1.000
  - 19 1.00111000 110 Y 1.010
  - 138 1.00011010 011 Y 1.001
  - 63 1.11111100 111 Y 10.000

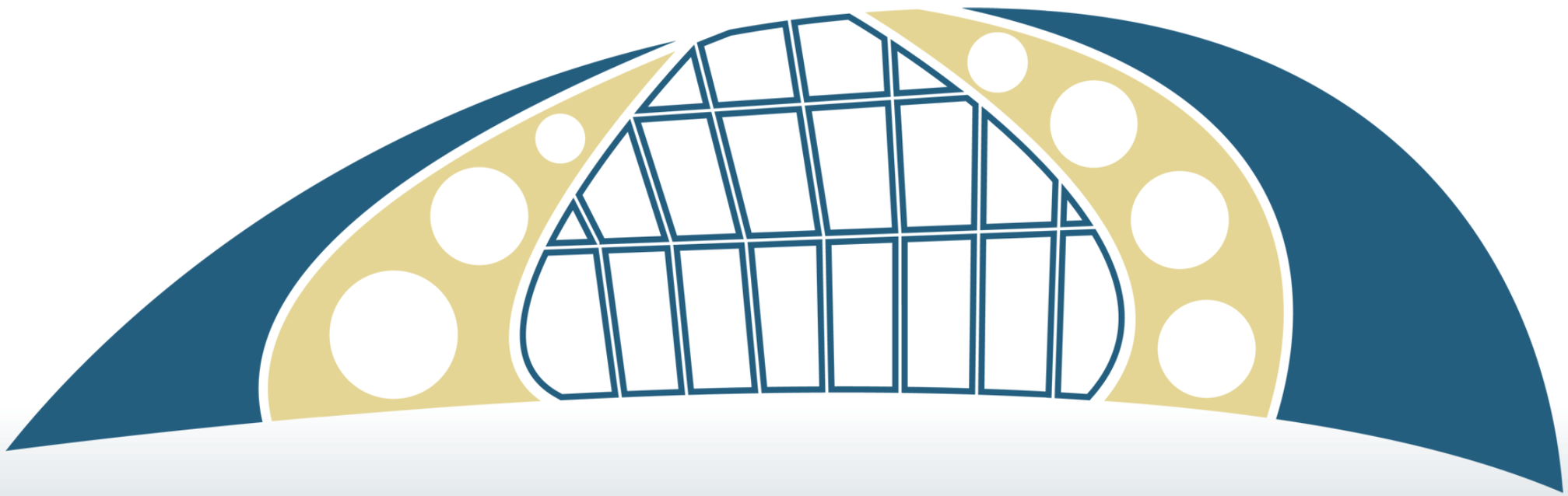
CREATING FLOATING POINT NUMBER

POSTNORMALIZE



- Issue
  - Rounding may have caused overflow
  - Handle by shifting right once & incrementing exponent

Value	Rounded	Exp	Adjusted	Result
128	1.000	7		128.0
15	1.101	3		15.0
17	1.000	4		16.0
19	1.010	4		20.0
138	1.001	7		134.0
63	10.000	5	1.000/6	64.0



WESTMONT INSPIRED

COMPUTING LAB